EFFECTS OF STRESS ON THE MAGNETIC PROPERTIES OF STEELS

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INTRODUCTION

As described in previous work [1], an applied uniaxial stress σ acts in some respects like an applied magnetic field operating through the magnetostriction λ. This additional "field" H_σ can be described by considering the energy A of the system along the reversible anhysteretic magnetization curve,

$$A = \mu_0 HM + \frac{\mu_0}{2}\alpha M^2 + \frac{3}{2}\sigma\lambda + TS \tag{1}$$

where T is temperature, S is entropy and $\mu_0\alpha M^2/2$ is the self-coupling energy. The dimensionless term α has been defined previously [2] and represents the strength of the coupling of the individual magnetic moments to the magnetization M. The effective magnetic field causes a change in magnetization, and therefore is determined by the derivative of this energy with respect to magnetization M. The derivative of entropy with respect to bulk magnetization M in a ferromagnet will be negligible in the cases under consideration because the fields applied here do not increase the ordering within the domains, although they do lead to a change in the bulk magnetization M. Therefore the effective field is given by

$$H_{eff} = \frac{1}{\mu_0}\frac{dA}{dM} = H + \alpha M + \frac{3}{2}\frac{\sigma}{\mu_0}\frac{d\lambda}{dM} \,. \tag{2}$$

This means that changes of the anhysteretic magnetization as a result of the application of stress can be calculated. In cases where the applied stress σ_0 is not coaxial with the direction along which λ and M are measured, the stress σ used in equation (2) is simply the component of applied stress along this direction. For isotropic materials this is given by

$$\sigma = \sigma_0 (\cos^2\theta - \upsilon\sin^2\theta) \tag{3}$$

where θ is the angle between the axis of the applied stress σ_0 and the axis of the magnetic field H, and υ is Poisson's ratio. Consequently H_σ, the component of the effective field due to stress, is

$$H_\sigma = \left(\frac{3}{2}\frac{\sigma}{\mu_0}\frac{\delta\lambda}{\delta M}\right)_\sigma = \frac{3}{2}\frac{\sigma_0}{\mu_0}\left(\frac{\delta\lambda}{\delta M}\right)_\sigma (\cos^2\theta - \upsilon\sin^2\theta) \tag{4}$$

Therefore, if the magnetostriction λ can be described as a function of magnetization and stress, then H_σ can be determined. The anhysteretic magnetization at field H and stress σ is identical to the anhysteretic at field $H+H_\sigma$ and zero stress,

$$M_{an}(H,\sigma) = M_{an}(H + \alpha M + H_\sigma, 0) = M_{an}\left(H + \alpha M + \frac{3}{2}\frac{\sigma}{\mu_0}\left(\frac{d\lambda}{dM}\right)_\sigma, 0\right) \tag{5}$$

where the effects of stress have been incorporated into the equivalent effective field.

This approach requires a description of the bulk magnetostriction, which depends on the domain configuration throughout the material. Theoretically if a certain domain configuration were assumed, this relationship could be determined via the known magnetostriction coefficients λ_{100} and λ_{111}. However, in practice this domain configuration in a material cannot be known in advance. It is therefore necessary to develop an empirical model to describe the relation between bulk magnetostriction and bulk magnetization. Since the magnetostriction must be symmetric about M = 0, a simple series expansion gives

$$\lambda = \sum_{i=0}^{\infty} \gamma_i M^{2i} \tag{6}$$

A reasonable first approximation to the magnetostriction of iron can be obtained by including the terms up to i=2. Ignoring the constant term, which is simply the elastic strain and does not play an active role in the magnetomechanical effect, this gives

$$\lambda = \gamma_1 M^2 + \gamma_2 M^4 \tag{7}$$

A more sophisticated approach to describing the magnetostriction curve, which includes hysteresis, has been given by Sablik and Jiles [3], but that approach will not be utilized in the present calculations. Improvements to the description of magnetostriction as a function of magnetization can also be achieved through the inclusion of higher order terms in equation (7).

Stress Dependence of Magnetostriction

The stress dependence of the magnetostriction curve $\lambda(M,\sigma)$ can be described from the stress dependence of γ_1 and γ_2. Using a Taylor series expansion,

$$\gamma_i(\sigma) = \gamma_i(0) + \sum_{n=1}^{\infty} \frac{\sigma^n}{n!} \gamma_i^n(0) \tag{8}$$

where $\gamma_i^n(0)$ is the nth derivative of γ_i with respect to stress at $\sigma = 0$. Using only the terms as far as n=1, and applying the above equation to the magnetostriction data of Kuruzar and Cullity [4], gave $\gamma_1(0) = 7 \times 10^{-18}$ $A^{-2}.m^2$, $\gamma_1'(0) = -1 \times 10^{-25}$ $A^{-2}.m^2.Pa^{-1}$, $\gamma_2(0) = -3.3 \times 10^{-30}$ $A^{-4}.m^2$, and $\gamma_2'(0) = 2.1 \times 10^{-38}$ $A^{-4}.m^4.Pa^{-1}$. The magnetostriction is then given by

$$\lambda = \sum_{i=0}^{\infty} \gamma_i(\sigma) M^{2i} \tag{9}$$

and the resulting effective field is obtained by substituting this into equation (3),

$$H_{eff} = H + \alpha M + \frac{3\sigma}{\mu_o} \sum_{i=0}^{\infty} i\gamma_i(\sigma) M^{2i-1} \tag{10}$$

$$= H + \alpha M + \frac{3\sigma}{\mu_o} \sum_{i=0}^{\infty} \left(i.M^{2i-1} \sum_{n=0}^{\infty} \frac{\sigma^n}{n!} \gamma_i^n(0) \right) \tag{11}$$

In the isotropic limit, the stress dependence of the anhysteretic magnetization curve can be determined from the equation

$$M_{an}(H,\sigma) = M_s \left\{ \coth\left(\frac{H+H_\sigma+\alpha M}{a}\right) - \frac{a}{H+H_\sigma+\alpha M} \right\} \tag{12}$$

Stress dependent anhysteretic magnetization curves from the measurement data of Jiles and Atherton [5] are shown in Fig. 1. The anhysteretic curves at various stress levels cross at different points which is a direct result of the stress dependent magnetostriction curve of iron $\lambda(M,\sigma)$. Calculations using a stress independent magnetostriction curve (i.e. with $\gamma_1(0) = 0$ and $\gamma_2(0) = 0$) have shown that all anhysteretics cross

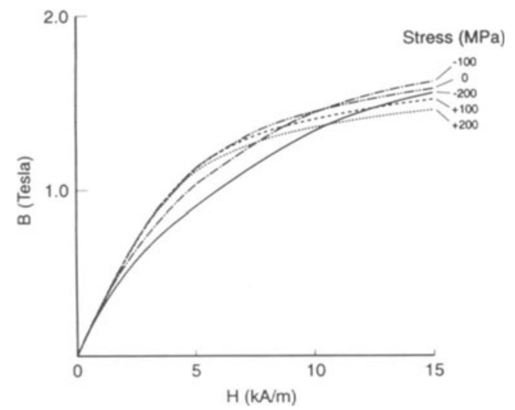

Fig. 1. Measured variation of the anhysteretic magnetization with stress, as reported by Jiles and Atherton [5].

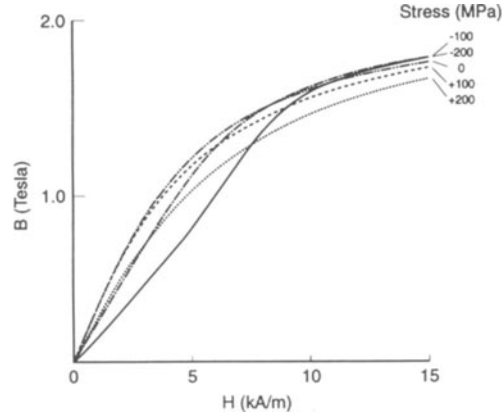

Fig. 2. Modeled variation of the anhysteretic magnetization curve for various levels of stress using equations (11) and (12) togetherwith the following values of the coefficients: $M_s = 1.7 \times 10^6$ A/m, a = 1000 A/m, k = 1000 A/m, $\alpha = 0.001$, c = 0.1, $\gamma_1 = 4 \times 10^{-18} - (2 \times 10^{-26})\sigma\ A^{-2}.m^2$ and $\gamma_2 = 2 \times 10^{-30} - (5 \times 10^{-39})\sigma\ A^{-4}.m^4$.

at the same location on the M,H plane. The predictions of the present model equation for the stress dependent anhysteretic are shown in Fig. 2 for selected values of the model parameters.

Stress Dependence of Magnetization

The effect of changing stress on the magnetization of a magnetic material leads to behavior in which the magnetization has been observed to increase or decrease when exposed to the same stress under the same external applied field. This indicates that the phenomenon is dependent on more than simply external factors of stress σ and magnetic field H. In fact the behavior depends on the magnetization history of the specimen which for major (i.e. symmetric) hysteresis loops can be expressed in terms of the displacement from the anhysteretic $M_{an} - M$. This, together with the field H and stress σ, specifies the state of the material on a major hysteresis loop.

Given these conditions, it has been found in previous studies [5,6,7], that the direction of the change in magnetization with applied stress is independent of the sign of the stress for small stresses when the magnetization is sufficiently distant from the anhysteretic. Therefore the direction of change is not directly dependent on the stress, but on some other related quantity which is independent of the sign of the stress. The elastic energy per unit volume W supplied to the material by the changing applied stress depends on the square of the stress,

$$W = \sigma^2/2E \tag{13}$$

where E is the relevant elastic modulus. It may reasonably be anticipated that this elastic energy causes unpinning of domain walls.

Reversible Component of Magnetization

In previous work [8] it has been shown that the reversible component of magnetization M_{rev} is given by

$$M_{rev} = c(M_{an} - M_{irr}) \tag{14}$$

where M_{an} is the anhysteretic magnetization and M_{irr} is the irreversible magnetization, which is achieved when all domain walls are returned to their planar condition and all reversible rotations of domain magnetizations are relaxed back to zero. The coefficient c, which has been defined previously [8], describes the flexibility of the magnetic domain walls. This equation can then be differentiated with respect to the elastic energy W supplied to the material as a result of applied stress.

$$\frac{dM_{rev}}{dW} = c\left(\frac{dM_{an}}{dW} - \frac{dM_{irr}}{dW}\right) . \tag{15}$$

Irreversible Component of Magnetization

In order to describe the irreversible changes in magnetization with stress we examine a law of approach applied only to the irreversible component of magnetization. This law can be expressed as

$$\frac{dM_{irr}}{dW} = \frac{1}{\xi}(M_{an} - M_{irr}) \tag{16}$$

where ξ is a coefficient with dimensions of energy per unit volume which relates the derivative of irreversible magnetization with respect to elastic energy to the displacement of the irreversible magnetization from the anhysteretic magnetization. The derivative of the total magnetization with respect to the elastic energy is then obtained by summing the irreversible and reversible components from equations (15) and (16).

$$\frac{dM}{dW} = \frac{1}{\xi}(M_{an} - M_{irr}) + c\frac{d}{dW}(M_{an} - M_{irr}) \tag{17}$$

$$= \frac{(1-c)}{\xi}(M_{an} - M_{irr}) + c\frac{dM_{an}}{dW} . \tag{18}$$

This last equation can be transformed into a derivative with respect to stress σ. From equation (13) the differential of the elastic energy dW is given by

$$dW = \left(\frac{\sigma}{E}\right) d\sigma \tag{19}$$

and therefore equation (18) becomes

$$\frac{dM}{d\sigma} = \frac{1}{\varepsilon^2}\, \sigma(1-c)(M_{an} - M_{irr}) + \frac{dM_{an}}{d\sigma} \tag{20}$$

where $\varepsilon = (E\xi)^{1/2}$ is a coefficient which has dimensions of stress.

Alternatively, using equation (14) and the expression $M = M_{irr} + M_{rev}$, equation (18) can be shown to be equivalent to

$$\frac{dM}{dW} = \frac{1}{\xi}(M_{an} - M) + c\frac{dM_{an}}{dW} \tag{21}$$

which conveniently expresses the law in terms of directly measurable quantities M and M_{an}.

RESULTS

The results of model calculations using equation (21) are shown subsequently. In Figs. 3 and 4 calculations have been made using parameters which describe the material used by Pitman. The similarity between these theoretical predictions and the experimental measurements can be seen by comparing the results with Fig. 5. These results show good agreement between calculation and measurement both in terms of the shapes of the curves and the numerical values. The results show that the model gives theoretical justification for the differences in sign of $dB/d\sigma$ which have been observed by others in the same material under identical external conditions of stress and magnetic field [9]. The reason for the differences in behavior under apparently identical conditions arises because of differences in the magnetic field exposure of the material giving it a different "magnetic history" under the same external conditions.

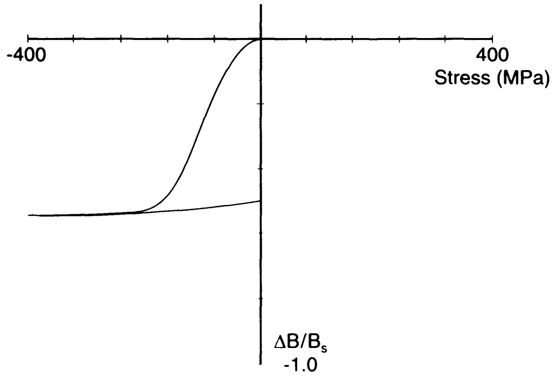

Fig. 3. Calculated variation of magnetic induction B with stress at a field of 80 A.m^{-1} under conditions similar to those employed by Pitman [9]. The specimen was first magnetized by applying a field of 40 kA.m^{-1} and the field was subsequently reduced to 80 A.m^{-1} . The specimen was then subjected to stress of up to 400 MPa. Values of the model parameters were: M_s = 1.71 x 10^6 A.m^{-1}, a = 955 A.m^{-1}, k = 2015 A.m^{-1}, α = 0.8 x 10^{-3}, c = 0.099, γ_{11} = 2 x 10^{-18} A^{-2}.m^2, γ_{12} = 1 x 10^{-26} A^{-2}.m^2.Pa^{-1}, γ_{21} = 1 x 10^{-30} A^{-4}.m^4, γ_{22} = 5 x 10^{-39} A^{-4}.m^4.Pa^{-1}, ε = 0.7 x 10^8 Pa, ξ = 24.5 x 10^3 Pa.

The calculated changes in magnetic induction at three different field strengths under conditions similar to those investigated experimentally by Craik and Wood in mild steel [6] are shown in Fig. 6. The results show an increasing amplitude of the magnetomechanical effect as the field was increased from 26 A.m^{-1} to 132 A.m^{-1} along the initial magnetization curve. The looping behavior under tension became more pronounced as the field amplitude was increased. This is in agreement with the experimental observations in Fig. 7. Furthermore, under compression the amplitude of the magnetomechanical effect was found to be much reduced, with at first an increase, but then a pronounced decrease in magnetic induction as the compressive stress was increased.

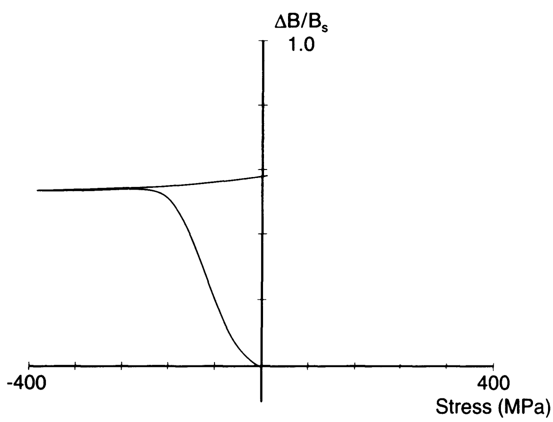

Fig. 4. Calculated variation of magnetic induction B with stress at a field of 80 A.m^{-1} under conditions similar to those employed by Pitman [9]. The specimen was first magnetized by applying a field of -40 kA.m^{-1} and the field was subsequently increased to 80 A.m^{-1}. The specimen was then subjected to stress of up to 400 MPa. Values of the model parameters were: M_s = 1.71 x 10^6 A.m^{-1}, a = 955 A.m^{-1}, k = 2015 A.m^{-1}, α = 0.8 x 10^{-3}, c = 0.099, γ_{11} = 2 x 10^{-18} A^{-2}.m^2, γ_{12} = 1 x 10^{-26} A^{-2}.m^2.Pa^{-1}, γ_{21} = 1 x 10^{-30} A^{-4}.m^4, γ_{22} = 5 x 10^{-39} A^{-4}.m^4.Pa^{-1}, ε = 0.7 x 10^8 Pa, ξ = 24.5 x 10^3 Pa.

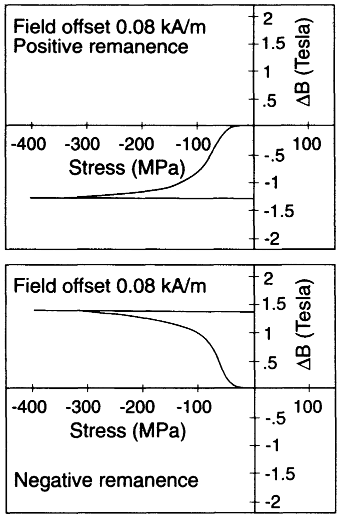

Fig. 5. Variation of magnetic induction B with compressive applied stress under an applied field of H = 80 $A.m^{-1}$ after Pitman [9]: (a) above the anhysteretic; and (b) below the anhysteretic.

These results give the first theoretical explanation for the changes in sign of $dB/d\sigma$, which have been observed, as stress is increased monotonically on some materials. This phenomenon has been widely observed in some iron alloys under compressive stress. The reason for this is that while the applied stress causes the prevailing magnetization to approach the anhysteretic magnetization, it also changes the value of the anhysteretic. Therefore as stress is continually increased the anhysteretic magnetization can actually cross the prevailing magnetization with a resultant change in sign of $dB/d\sigma$ as the stress increases further. A specific example occurs in materials with positive $\frac{d\lambda}{dM}$ when subjected to increasing compressive stress.

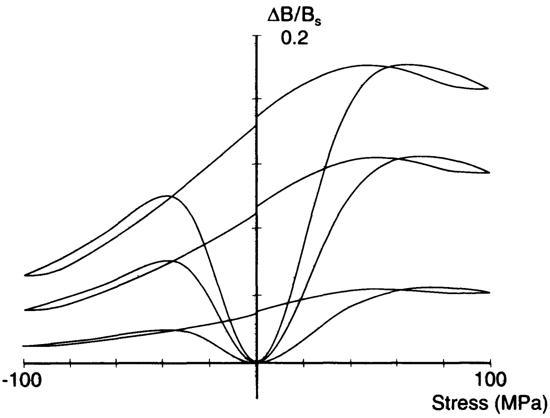

Fig. 6. Calculated variation of magnetic induction B with stress at fields of 26 $A.m^{-1}$ (lower), 80 $A.m^{-1}$ (middle) and 132 $A.m^{-1}$ (upper) under conditions similar to those employed by Craik and Wood [6]. The specimen was demagnetized and then subjected to a field of the given magnitude. It was then subjected to an applied stress of up to 100 MPa, either in tension or compression. Values of the model parameters were: $M_s = 1.71 \times 10^6\ A.m^{-1}$, $a = 900\ A.m^{-1}$, $k = 2000\ A.m^{-1}$, $\alpha = 1.1 \times 10^{-3}$, $c = 0.1$, $\gamma_{11} = 2 \times 10^{-18}\ A^{-2}.m^2$, $\gamma_{12} = 1.5 \times 10^{-26}\ A^{-2}.m^2.Pa^{-1}$, $\gamma_{21} = 2 \times 10^{-30}\ A^{-4}.m^4$, $\gamma_{22} = 5 \times 10^{-39}\ A^{-4}.m^4.Pa^{-1}$, $\varepsilon = 1.1 \times 10^7$ Pa, $\xi = 605$ Pa.

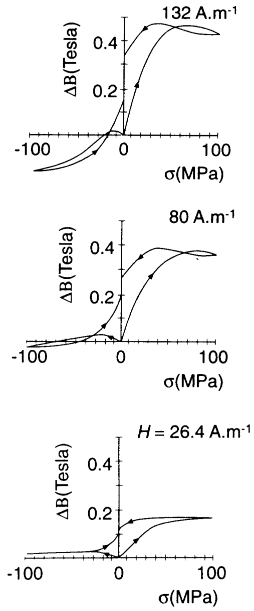

Fig. 7. Variation of magnetic induction B with stress for a specimen of mild steel, after Craik and Wood [6]. At low stress amplitudes the change in magnetization with stress has the same sign, irrespective of whether the stress is compressive or tensile. This indicates that $M_{an}(H,\sigma) - M(H,0)$ dominates the process at low stress. At compressive stresses beyond -30 MPa the stress derivative $dB/d\sigma$ changes sign, indicating that the magnetization has crossed the anhysteretic.

CONCLUSIONS

The model theory described in this paper has been developed to explain the apparently disparate range of observations of the magnetomechanical effect that have been reported previously in the literature. The equations have been derived based on the concept that under a changing applied stress at constant magnetic field, the magnetization changes so that it approaches the anhysteretic magnetization. This concept has been developed to include a quantitative description of stress-dependent magnetostriction and anhysteretic magnetization curves, and the mechanism by which the change in elastic energy supplied to the material causes a reduction in the displacement of the magnetization from the anhysteretic magnetization.

ACKNOWLEDGMENT

This work was supported by the U.S. Department of Energy, Office of Basic Energy Sciences under contract no. W-7405-Eng-82.

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